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ERASE: AN OVERVIEW

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Prepared for

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Lincoln Laboratory

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FOR THE COMMANDER

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## ABSTRACT

The defense resource allocator computer programs ERASE is described herein. Given a MaRV with penetration aid attack scenario, the total radar resources available, and the number and characteristics of the defense interceptor, ERASE allocates the radar and interceptor resources on a continuing basis as the attack unfolds.

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## 1. INTRODUCTION

This overview to ERASE (Engagement Resource Allocation Simulator and Evaluator) is written to provide a non-technical overview of the intent, logical flow, and operation of the program. It also serves as a high-level technical summary of the procedures used by ERASE to simulate an optimal defense against a MaRV attack. These purposes are, in a sense, contradictory, therefore the report has been organized for multiple uses. Section 1.1 contains the high level overview of ERASE. Section 1.2 contains the next level of detail. Sections 2 through 5 contain many technical details on ERASE procedures and should be skipped on first reading by those not needing these mathematical details. Section 6 contains a simple example that can be used to illustrate the operation of the program.

### 1.1 OVERVIEW OF ERASE

Defense against a MaRV threat is not static. Variation of net size with altitude and ballistic aim point, multiple discriminants, and diverse penails require a general, sophisticated, and adaptive allocation of defense resources, which comprise interceptors, radar power, and battlespace (time). ERASE simulates a MaRV engagement, and performs this resource allocation on an object by object basis. The user can specify the defense (radar, interceptor nets, discrimination capability) and the offense (numbers of MaRVs and decoys, decoy characteristics, threat tube structure). The simulation then returns statistics of the engagement from which defense (or offense) performance can be comprehensively measured. ERASE currently provides a fairly general simulation of defense resource allocation. The adaptive nature of the allocation of defense resources allows great flexibility, since new allocation templates need not be designed for each new offense configuration. Generality derives from the large number of parameters available as input, allowing the user to test a broad number of engagements, discriminants, RV's, decoys, and the other penails, particularly chaff. The use of probability density functions (rather than just mean and variance) to characterize performance and object by object allocation gives the designer a wealth of data from which to analyze why a particular penail design performed as it did.

Experience with the simulation has shown that in all but the most clear cut of cases, adaptive allocation is required to achieve the best defense performance. The cases in which template allocation is acceptable are those in which either the offense overwhelms the defense (with either a few perfect decoys or a very large number of traffic decoys) or the defense is impenetrable (a vast stockpile of excellent interceptors or an enormous radar). Some crucial features of the engagement are chaff (to reduce available battlespace), interceptor stockpile and quality (the most important engagement parameters in most cases), and a balanced decoy design. Given adaptive allocation, the defense will exploit an imbalance in decoy quality (say a good exo-atmospheric match but a poor slowdown match.)

Figure 1a is a schematic of an engagement. Chaff is modelled as completely masking the threat above the altitude  $H_{\text{CHAFF}}$ . Below  $H_{\text{CHAFF}}$ , search begins. During processing, objects are detected, verify pulses are sent (to remove potential false detections), and then track commences. Discrimination is flexibly modelled, so that arbitrary or multiple discriminants (defined by altitude band) can be handled. The resource allocator chooses the best partial net and commit altitudes of the interceptors.

ERASE inputs are: parameters which model discrimination, firing doctrine, radar, threat, and stockpile. The resource allocator manipulates the parameters shown in Figure 1b: the probability mass function for interceptor usage (the probability that there are  $I$  interceptors left at the end of the engagement), the probability mass function for penetrators (the probability that  $N$  RV's penetrate the defense) and a distribution of radar power with engagement time, by category (search, verify, track, discrimination). Not shown in Figure 1b are additional outputs that can be obtained: the probability density function for damage inflicted on the target, the distribution of computer resource usage with engagement time, and distributions of altitudes at which objects are engaged.



Figure 1a  
ENGAGEMENT

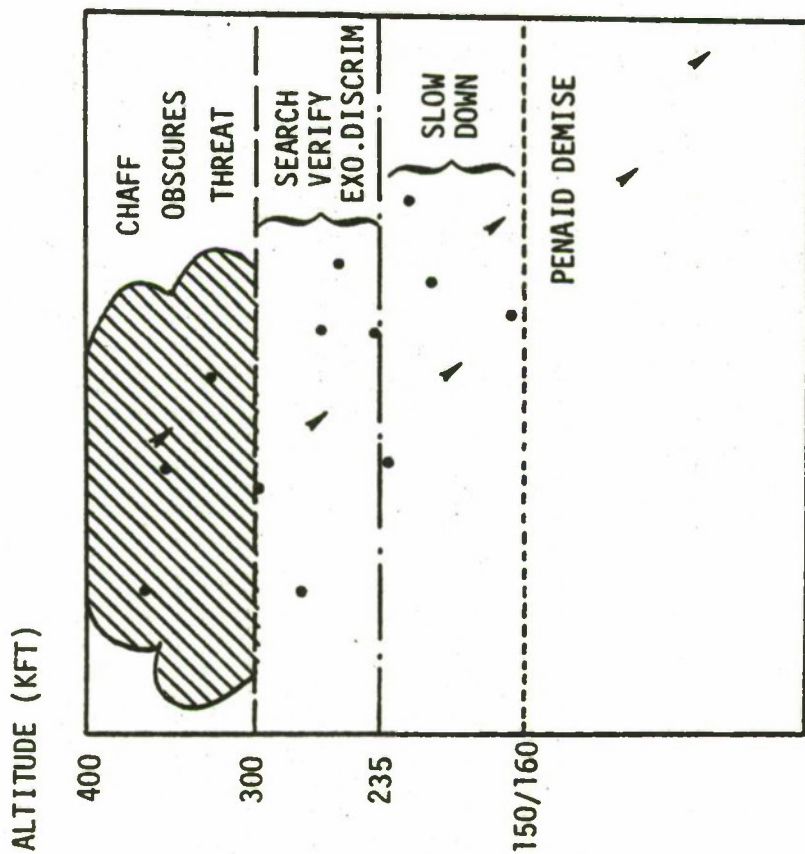


Figure 1b  
SIMULATION STRUCTURE

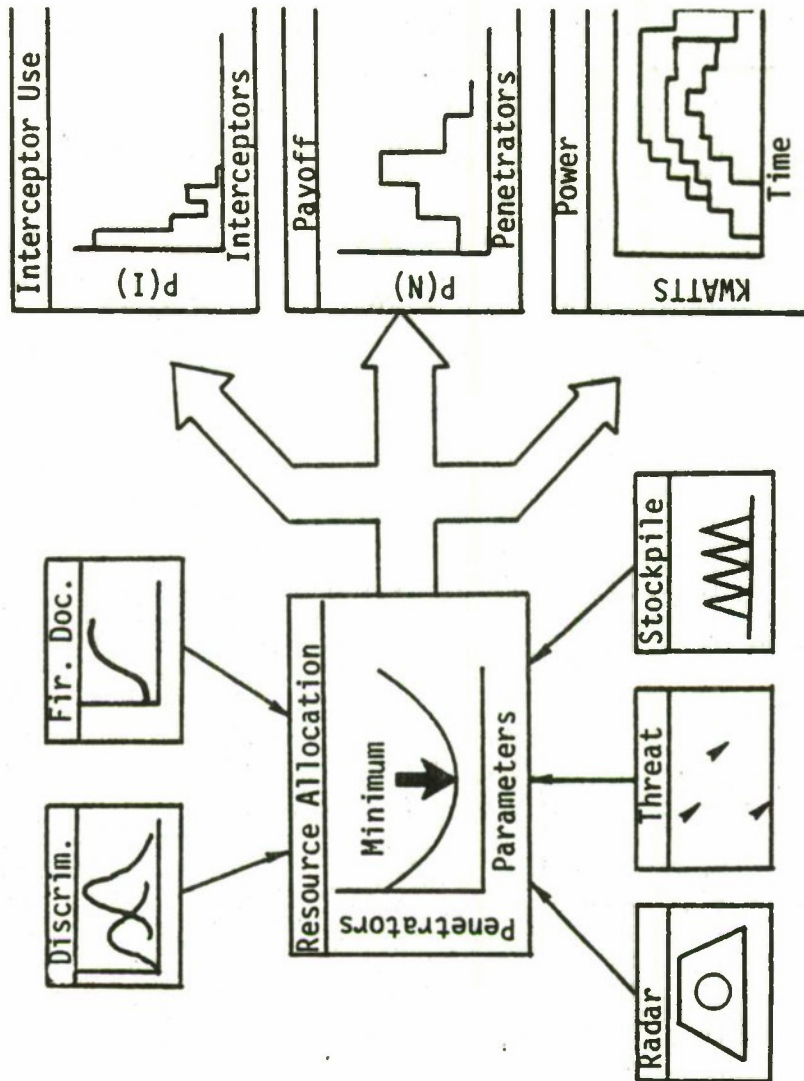


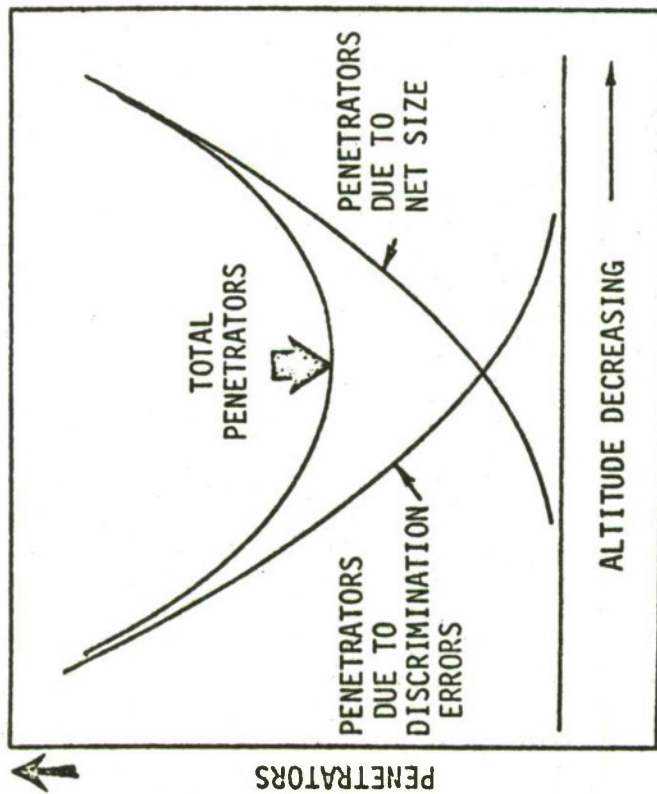
Figure 1



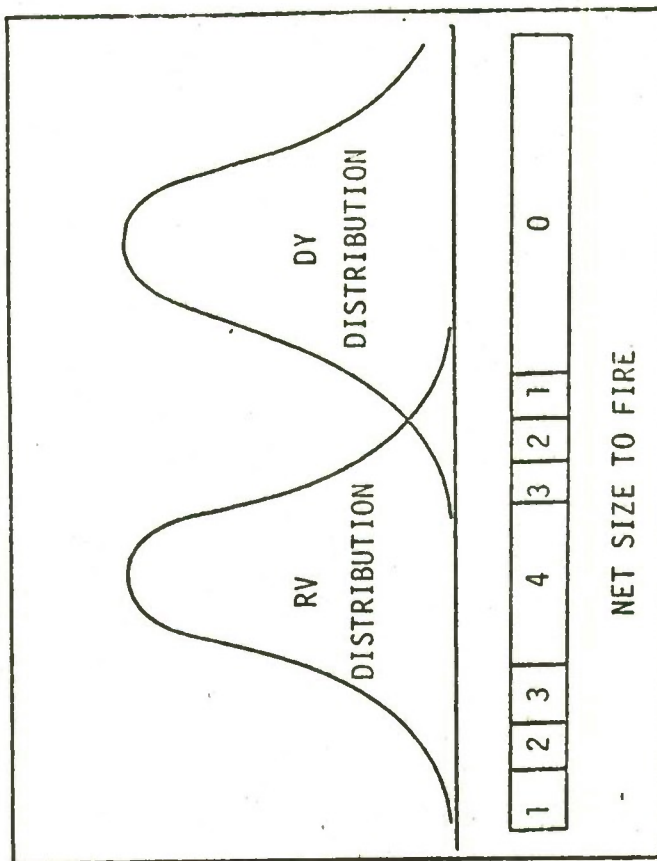
#### 1.1.1 The Primary Tradeoffs Considered by ERASE

The primary tradeoff performed by the resource allocator is between discrimination errors and interceptor net size. When objects are first detected (high altitude), the defense does not know which are RV's and which are decoys; on the other hand, the interceptor net size required is small. If the stockpile were large, the defense could fire this small net at each object detected. For a realistic stockpile, however, there are not enough interceptors to waste even small nets on each decoy, hence the defense must take discrimination measurements to uncover the decoys in order to fire interceptors only at RV's. The tradeoff is made sharper since the required net size for a given kill probability increases with decreasing altitude; thus while taking measurements, the defense is losing stockpile effectiveness. Figure 2a shows schematically how the number of penetrators due to discrimination errors decreases with altitude because discrimination measurements are gathered on the objects, whereas the number of penetrators due to increasing net size increases. The allocator chooses the optimum commit altitude and discrimination rates to minimize the total number of penetrators without violating either the power constraint on the radar or the actual interceptor stockpile.

The resource allocator assigns interceptors by a multiple threshold firing doctrine, as shown in Figure 2b. The number of interceptors fired depends, among other things, on how certain the allocator is that a given object is an RV. For returns observed near the center of the RV distribution, the largest net is fired, since the probability that the object is an RV is very high, and the probability that it is a decoy is low. At the edges of the figure, one or no interceptors would be fired, since these returns would likely be from a decoy.



(a) Decision Altitude



(b) Firing Doctrine

Figure 2

### 1.1.2 Critical Defense Models

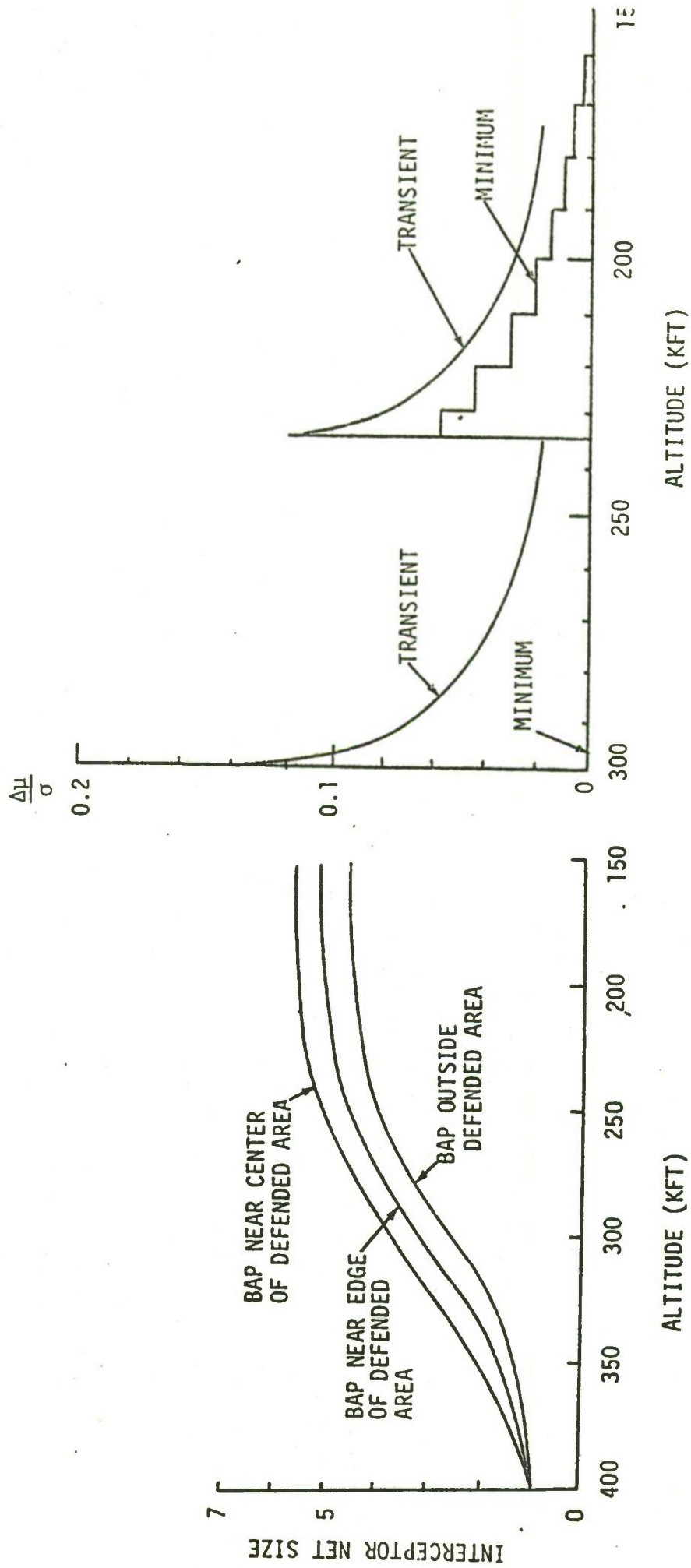
Figure 3a shows the net size required as a function of altitude for three representative ballistic aim points (BAP's). This models the effectiveness of the interceptor against MaRV maneuvers. Fewer interceptors are required for BAP's away from the center of the defended area than for BAP's near the center. This is very important for defense planning since considerable savings in interceptors can be made by modifying the allocation according to object BAP.

Figure 3b illustrates the model used for discrimination. Uncertainty due to each discriminant (characterized by models of the probability density functions for normalized discriminant algorithm outputs) is separated into three components, each of which is modeled by a function whose parameters are specified in a series of altitude bands, over which the parameters are held constant. The first component is uncorrelated measurement noise; the second is uncertainty that cannot be eliminated by measurements (such as manufacturing uncertainties between the objects); the third is correlated measurement noise (whose effect on discriminant uncertainty depends on measurement rate). The first two are illustrated in the figure. The change in transient part is due to a modeling change at 235 kft, whereas the minimum value of  $\Delta\mu/\sigma$  represents the error that cannot be removed by measurements.

## 1.2 SUMMARY OF ERASE LOGIC

The radar and interceptor resources in a MaRV BMD engagement can be allocated by the defense to minimize the expected number of penetrating warheads (leakers) during the course of an engagement. The purpose of ERASE (Engagement Resource Allocation Simulator and Evaluator) is to provide a model to describe the performance of such defense of an area target. ERASE is not a simulation in the Monte Carlo sense, since random numbers are not drawn and defense actions are not simulated in detail. ERASE is a statistical simulation model of performance, i.e., the statistics of performance and defense actions are gathered at each simulated event and combined to produce an overall statistical description of defense performance. This distinction is important for understanding the way ERASE operates and the source of its computational efficiency.





(a) Interceptor Net Size vs. Altitude and BAP

(b) Discrimination Description

Figure 3



The operation of ERASE is designed around a mathematical optimization problem; this optimization problem is the one faced by the defense in attempting to minimize the expected number of penetrating warheads. ERASE can be understood better by relating the optimization problem to ERASE procedures.

The problem faced by the defense is to orchestrate all its resources and capabilities to achieve its desired end: minimum leakage. A macroscopic view of this allocation problem is that the defense must allocate two types of resource over the engagement period to minimize the expected number of penetrators; these two resources are sensor and weapon related. The two resources allocated by the defense, and ERASE, are radar power, which is renewable, and interceptors, which are non-renewable. Radar power is allocated by specifying a sequence of waveforms to objects without specific pulse scheduling. The radar functions are categorized by pulse/waveform as follows:

- search
- verify
- early track
- discrimination
- final track.

The interceptor resources are allocated by specifying:

- net sizes
- commit times
- objects to be intercepted.

Within this context the mathematical optimization problem faced by the defense is:

minimize  $E[\text{Number of Penetrating Warheads}]$   
over{all possible power allocations to objects, object  
commit times, interceptor allocations}

This can be restated in a hierarchical form by nesting optimization as follows:

$$\begin{array}{ccccccc} \text{Minimize[} & & \text{minimize[} & & \text{minimize[} & & \text{minimize[Expected Leakage]} \\ \text{over} & \left\{ \begin{array}{l} \text{all power} \\ \text{allocations} \\ \text{to categories} \end{array} \right\} & \left\{ \begin{array}{l} \text{all power} \\ \text{allocations} \\ \text{to objects} \\ \text{by category} \end{array} \right\} & \left\{ \begin{array}{l} \text{object commit} \\ \text{times} \end{array} \right\} & \left\{ \begin{array}{l} \text{interceptor} \\ \text{allocations} \end{array} \right\} & & \end{array}$$

ERASE models a defense that solves its optimization problem by formulating it in the above way. The defense modelled by ERASE does not solve the true optimization problem, but solves a related problem, i.e., the solution procedure approximates the optimal allocation and optimal defense performance.

ERASE finds approximately optimum allocations of the defense resources by performing three basic hierarchical levels of optimization, nested as given in the above mathematical optimization problem. These three nested optimization levels are:

1. Category Allocation (Radar Power by Function)
2. Discrimination Allocation (Discrimination Power to Objects) and Commit Time Selection (Commit Times for Objects)
3. Interceptor Allocation (Firing Doctrine applied to Objects).

These allocators are run to find a defense operating point, where this operating point is determined by 3 nested optimization procedures. The dynamic defense strategy is composed of the operating points (resource allocations) committed at each point in time by ERASE. As in any simulation, ERASE cannot afford to compute an operating point at all times, but can afford to compute operating points at discrete points in time chosen close enough together that the allocation of resources selected for the interval between recalculation remains almost optimal. This is the procedure used by ERASE; events are defined and resource allocations remain constant between events. At event times a complete reallocation is performed, involving all 3 levels of optimization, to find the best operating point to use in the next inter-event interval. The events specified in ERASE are

- (1) Times at which any object changes radar processing categories (search, verify, ...)
- (2) Times at which any object is tentatively scheduled to have interceptors committed to it, and
- (3) Other times inserted to insure a good approximation to continual review of resource allocations.

As indicated in Figure 4, ERASE consists of a simulation portion and an evaluation portion. The two parts of ERASE are logically separate for the following reason. The defense modelled in the simulation makes decisions that may or may not be optimal based on models of reality of varying fidelity. This defense makes decisions that ultimately lead to a simple summary of defense actions:

- (1) each object's commit time and altitude
- (2) each object's discrimination history at that time (the  $P_E$  vs  $P_{FA}$  tradeoff curve) and
- (3) each object's interceptor allocation, given explicitly or implicitly by a firing doctrine rule.

This information is passed to the evaluator which scores the engagement by computing the statistics of

- (1) interceptor usage, and
- (2) RV penetration.

The output of the entire process is then a statistical summary of defense performance and a summary of defense allocation decisions.

This summary of ERASE's allocation procedures is expanded in subsequent sections.



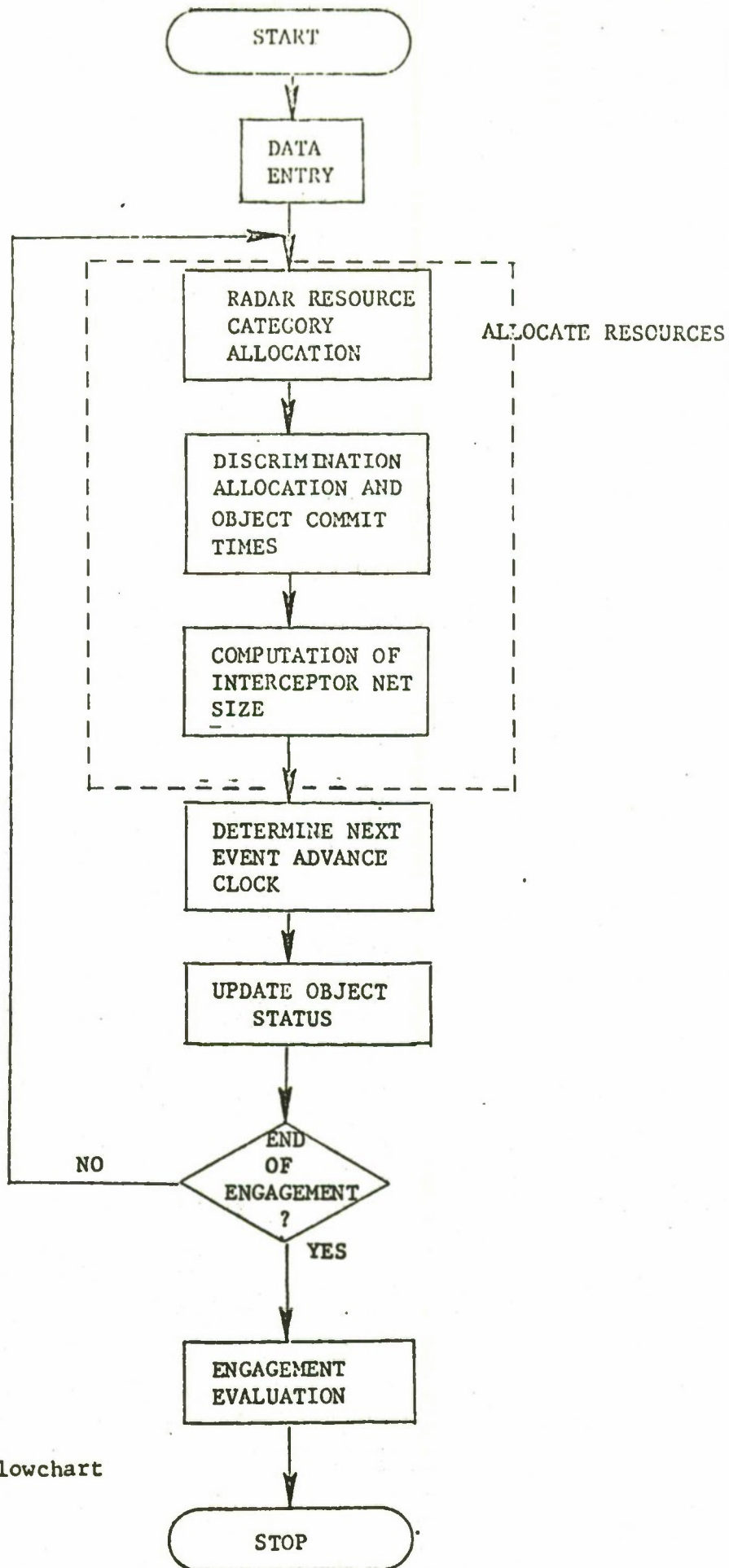


Figure 4. ERASE Flowchart



## 2. CATEGORY ALLOCATION

The category allocator divides the available radar power among the three functional categories (search, early track, and discrimination) in order to minimize an estimate of the total time required to process an average object from the time it enters the search sector until interceptors are committed to it. The other functions, verify and final track, are performed at fixed rates, and hence sufficient power for these functions is simply set aside.

The category allocator is run at each event in the simulation before the discrimination allocation and commit time and the interceptor allocation optimization procedures are run. The criterion minimized in the category allocator, processing time for the typical object, is not the same criterion as in the exact mathematical defense problem, however it is a reasonable criterion to choose: it decouples this allocation problem from the inner optimizations. This implies that the resource allocation procedures can be executed effectively. The implication of allocating power to minimize processing time is that objects will be processed at as high an average altitude as possible; the high altitudes have lower nominal nets, hence the scarce interceptors can be used to achieve high kill probabilities.

The expression for the variable processing time  $T$  is

$$T = \frac{k_s}{X_s} + \frac{k_{ET}}{X_{ET}} + \frac{k_D}{X_D}$$

where

$k_s$  = average number of search pulses sent before object detection. ( $k_s = \frac{1}{2} b$  where  $b$  is the number of beams in search sector\*)

---

\*It is assumed that the probability of an object entering any particular beam of the search sector is uniform over the entire search sector. The average number of beams scanned before the object is detected is  $\frac{1}{2}b$  and  $\frac{\frac{1}{2}b}{X_s}$  is then the average time.

$x_s$  = search pulse (scan) rate

$k_{ET}$  = required number of pulses to complete early track processing

$x_{ET}$  = early track pulse rate

$k_D$  = average number of pulses to complete discrimination processing for objects in discrimination

$x_D$  = average discrimination pulse rate.

The minimization of the performance index  $J$  is constrained by the radar power available  $R$ ,

$R$  = resource available (power) after items such as verify and final track are accounted for

$\alpha_s$  = search pulse energy

$\alpha_{ET}$  = early track pulse energy

$\alpha_D$  = average discrimination pulse energy

$N_{ET}$  = number of objects in early track

$N_D$  = number of objects in discrimination.

Now the power allocated is:

- Search:  $\alpha_s x_s$
- Early track:  $\alpha_{ET} x_{ET} N_{ET}$
- Discrimination:  $\alpha_D x_D N_D$ .

Other considerations introduce additional constraints on pulse rates. For example, a certain minimum track rate may be necessary to prevent losing track of objects. In this problem it is assumed that minimum rates are specified for search, track, and discrimination. The task rates may be written in the form:

$$x_S = (x_S)_{\min} + \Delta x_S$$

$$x_{ET} = (x_{ET})_{\min} + \Delta x_{ET}$$

$$x_D = (x_D)_{\min} + \Delta x_D$$

Under these assumptions the performance index to be minimized is:

$$T = \frac{k_S}{(x_S)_{\min} + \Delta x_S} + \frac{k_{ET}}{(x_{ET})_{\min} + \Delta x_{ET}} + \frac{k_D}{(x_D)_{\min} + \Delta x_D}$$

The resource constraint is:

$$\begin{aligned} & \alpha_S \Delta x_S + \alpha_{ET} \Delta x_{ET} N_{ET} + \alpha_D \Delta x_D N_D \\ & = R - \alpha_S (x_S)_{\min} - \alpha_{ET} (x_{ET})_{\min} N_{ET} \\ & \quad - \alpha_D (x_D)_{\min} N_D, \end{aligned}$$

where the control variables have been grouped on the left-hand side of the constraint equation. The control variables are constrained to be positive.

The optimization problem described above may be solved explicitly using standard Lagrange multiplier techniques.

The result of the category allocator is to distribute total radar power to the radar functions. The power is then allocated to objects as follows:

### Search

Radar search power is allocated to send pulses up beams in the search sector. Since each beam is assumed equally likely to contain an object, they are scanned sequentially with each one receiving a pulse in turn. The pulse rate is determined by  $X_s$ .

### Verify

Each object in verify receives pulses at the predetermined rate.

### Early Track

The object by object allocation of radar power within early track is accomplished by allocating the same amount of power to each object in early track since at this stage of processing each object is assumed equally likely to be a warhead. Thus each object receives  $\alpha_{ET} X_{ET} \text{ kw}$ , i.e., pulses at rate  $X_{ET}$ .

### Final Track

Each object in final track receives pulses at the prespecified rate.

### Discrimination

Discrimination power is allocated dynamically to objects by the discrimination allocator described in the next section.



### 3. DISCRIMINATION ALLOCATOR AND COMMIT TIME SELECTOR

Discrimination is the most important radar function which must be allocated on an object by object basis. The purpose of discrimination resource allocation is to distribute the discrimination power available among the objects in discrimination to achieve the minimum possible expected number of penetrators. Thus allocation of this resource must be coupled with the commit time decision and the interceptor allocation (firing doctrine) decision. This section contains discussion of the discrimination resource allocation and the selection of commit times for objects in discrimination.

Recall that, with the category allocation already decoupled and performed, the mathematical defense problem can be written as

$$\begin{aligned}
 & \text{minimize} \quad [ \quad \min \quad [ \min \quad [\text{Expected Leakers}]] ] \\
 & \left\{ \begin{array}{l} \text{possible discrimination} \\ \text{resource to objects} \end{array} \right\} \left\{ \begin{array}{l} \text{commit times} \\ \text{to objects} \end{array} \right\} \left\{ \begin{array}{l} \text{interceptor} \\ \text{allocations} \end{array} \right\} \\
 & = \min_{\{\text{discr.}\}} \min_{\{\text{time}\}} \min_{\{\text{inter.}\}} [ \sum_{\text{objects}} \text{Prob}\{\text{RV penetrates given decisions}\} ]
 \end{aligned}$$

The allocation problem is solved as follows. ERASE postulates assigning various discrimination pulse rates to each object; the postulated future pulse rate histories can be summarized by the pair  $\{r, t_c\}$  where this means that the object is scheduled to receive discrimination pulses at rate  $r$  until time  $t_c$ , at which time interceptor(s) may be launched. Note that ERASE restricts attention to constant pulse rate futures rather than general functions, however the decisions made are reviewed frequently enough to allow variable rates.

Graphically and logically the problem which ERASE solves approximately is (see Figure 5):

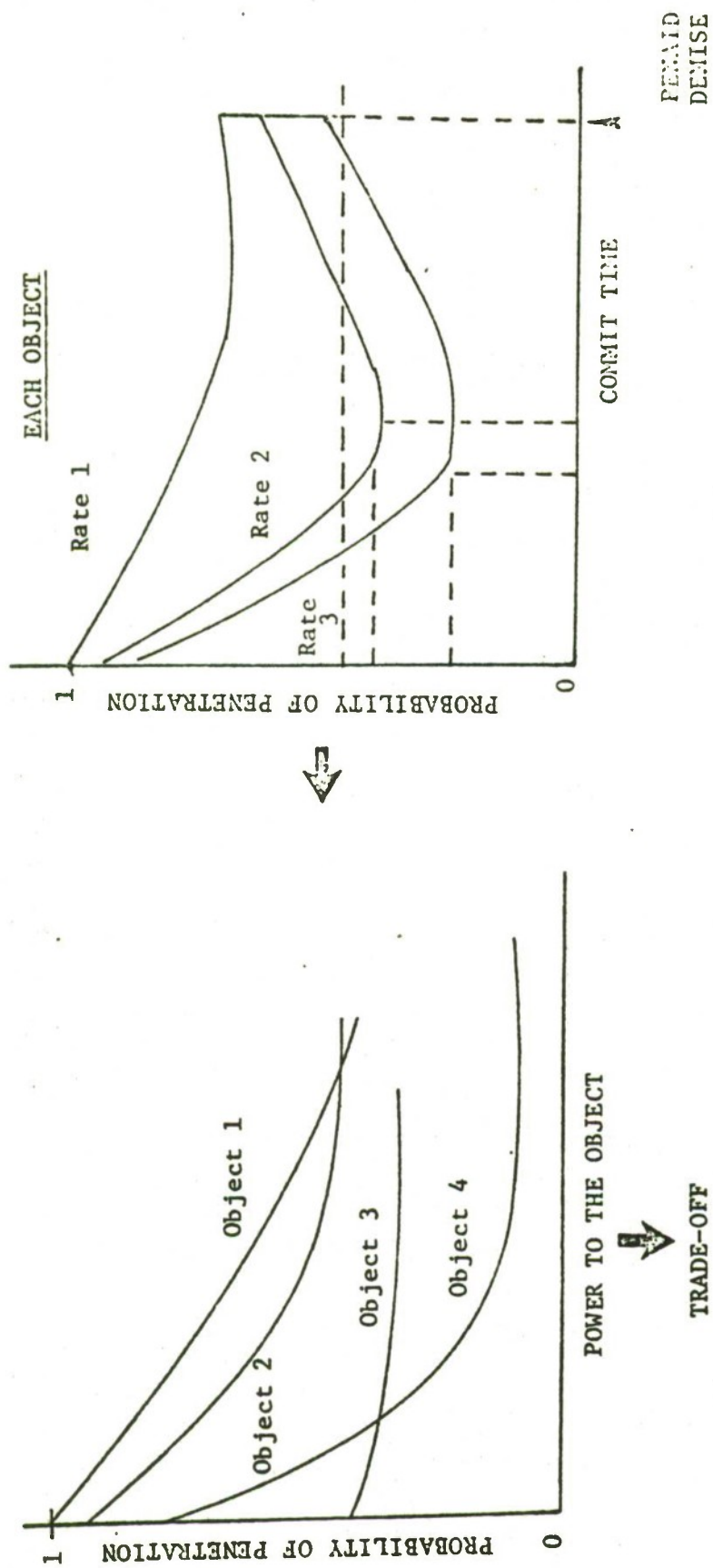


Figure 5. Discrimination Allocation and Commit Time Selection

- (1) Calculate the probability of penetration for each object for a variety of discrimination pulse rates and select the commit time for each rate which yields minimum probability of penetration; associate this minimum with that pulse rate.
- (2) Calculate the relationship between discrimination resource (power) and probability of penetration for each object.
- (3) Trade-off power vs. performance for all objects.

Note that in (1) above the probability of penetration calculations assume an optimal interceptor allocation will be made at the commit times  $t_c$ . This is the subject of the firing doctrine discussed in the next section. Mathematically the discrimination resource allocation problem solved by ERASE can be stated as the following optimization problem:

$$\text{Min} \left\{ \sum_{i=1}^n P^*(i; r_i) \right\} \text{ subject to } \sum_{i=1}^n p_i(r_i) \leq D$$

$(r_1, r_2, \dots, r_n)$

where

$r_i$  = the discrimination measurement rate assigned to object  $i$

$p_i(r_i)$  = power consumed in sending discrimination at object  $i$  at rate  $r_i$

$n$  = number of objects being discriminated

$D$  = discrimination power available ( $= \alpha_D X_D N_D$ ) from the category allocator

$P^*(i; r_i)$  = minimum penetration probability attainable (over all possible commit altitudes) if discrimination is done at rate  $r_i$  on object  $i$  consistent with interceptor stockpile constraints.



Solution of this problem exactly at each event is computationally expensive due to the large number of events in a realistic simulation and the complexity of the optimization algorithm. Thus ERASE performs the exact detailed allocation at selected times and performs an approximately optimal allocation using a priority list, described later, at all other event times. The times for the exact allocation are chosen so that between exact allocations

- the number of objects in discrimination remains relatively constant
- the power available remains relatively constant
- the power consumed by the assigned rates remains relatively constant
- the optimal commit time for an object is not passed without action being taken

The following is the method used to calculate the time of the next complete discrimination allocation update in light of these objectives.

$T_0$  = clock time of current complete discrimination resource allocation

$\Delta T_1$  =  $1/4$  (Average time to complete discrimination for the 5 objects in the last 10 to complete discrimination with the shortest discrimination time)

$\Delta T_2$  =  $1/4$  (The time for any object to achieve  $\Delta\mu/\sigma_{RB} = 4$  if discriminated at the highest rate now being given)

$\Delta T$  = step size =  $\max\{\Delta T_1, \Delta T_2\}$

$T$  = update time for next complete discrimination resource allocation



$$= \min\{T_0 + T, \text{earliest commit scheduled}, T_0 + 0.5\}$$

The exact solution to the discrimination allocation problem described above requires the functions  $P^*(i; r_i)$ , the minimal possible penetration probability for each object as a function of the discrimination measurement rate assigned. These can be found for any fixed object and rate  $(i; r_i)$  by determining the optimal commit time and optimal interceptor allocation at that commit time.

Given the object history (i.e., its descent velocity and the statistics of its past discrimination returns), the object future discrimination rate  $(i; r_i)$ , and any possible future commit time, the optimal interceptor allocation is given by the multiple threshold firing doctrine (see next section). Given this firing doctrine algorithm, one has

$$\begin{aligned} P^*(i; r_i) &= \min_{T_0 \leq t} \{P_t(i; r_i, \text{history})\} \\ &= P_{t_i^*}(i; r_i, \text{history}) \end{aligned}$$

The commit time  $t_i^*$ , and hence  $P_i^*(i; r_i)$ , can be found using various search algorithms. The algorithm used by ERASE to find  $t_i^*$  is a grid search. Under the assumption (verified empirically in selected cases) that  $P_t$  is unimodal except for a break at the decoy unmask altitude, the grid search is optimal if the grid is fine. The equation for  $P^*(i; r_i)$  then becomes

$$P^*(i; r_i) = \min_{t \in T} \{P_t(i; r_i, \text{history})\}$$

$$T = \{T_0 + \Delta T, T_0 + 2\Delta T, T_0 + 3\Delta T, T_0 + 4\Delta T, T_0 + 5\Delta T, T_{\text{unmask}}\}.$$

With the above simplifications, the resource allocation problem can be treated directly. The problem at a discrimination resource allocation update time is to find:

$$\text{Min } \left\{ \sum_{i=1}^n P^*(i; r_i) \right\} \quad \text{subject to } \sum_{i=1}^n p_i(r_i) \leq D.$$

$$(r_1, r_2, \dots, r_n)$$

This problem can be solved completely and generally using dynamic programming techniques.

If the  $P^*(i; r_i)$  are convex in  $r_i$ , which empirically is approximately true,\* the dynamic programming algorithm collapses to the simpler marginal return algorithm, which is used to solve the above problem. With the functions being convex, the optimal allocation has property that

$$\frac{dP^*(i; r_i^*)}{dp_i(r_i^*)} = \frac{dP^*(j; r_j^*)}{dp_j(r_j^*)}$$

for all  $i$  and  $j$  such that  $r_i^* \neq 0, r_j^* \neq 0$ . The solution to this  $n$  dimensional continuous rate problem can be approximated by solving the discrete rate problem by restricting the  $r_i$  to a grid of allowable rates:  $r_i \in R = \{R_1, R_2, \dots, R_m\}$ . The discrete solution approximates the exact solution to the extent that the grid in  $R$  is fine;  $R$  is an input parameter to ERASE. The algorithm is as follows:

- (1) Generate the matrix  $X = (x_{ik})$  for  $1 \leq i \leq n$  and  $1 \leq k \leq m-1$  as follows:

$$x_{ik} = \frac{P^*(i; R_k) - P^*(i; R_{k+1})}{p_i(R_{k+1}) - p_i(R_k)}$$

\*If a  $P^*(i; r_i)$  is not convex, the convex hull or other convex approximation (as used in ERASE) can be used without injecting excessive error.

- (2) Generate the matrix  $Y=(y_{ik})$  such that  $y_{ik}$  is the rank of  $x_{ik}$  sorted in descending order; i.e., if  $y_{ik}=1$ ,  $x_{ik}$  is the largest value in  $X$ . (Note that convexity of  $P^*(;)$  and linearity of  $p_i()$  insure that  $y_{ik} < y_{i, k+1}$ ).
- (3) Allocate the discrimination resource by
- (a) Assigning all objects a rate  $R_1$  (if  $\sum_{i=1}^n p_i(R_1) > D$ , no feasible solution exists) and
  - (b) iteratively cycle through the matrix  $Y$ , starting with the index  $(i,k)$   $y_{ik}=1$ , so that at each step if  $y_{ik}$  has the next smallest value, increase the rate assigned to object  $i$  from  $R_k$  to  $R_{k+1}$  if  $\sum_{i=1}^n p_i(r_i) \leq D$  after this assignment. If the power constraint would be violated, the algorithm terminates.

As new objects enter discrimination or conditions change, either a complete discrimination allocation update is required or a method is needed to approximate what such an update would dictate. At events when a complete update is not done, ERASE approximates a complete update by creating a new  $Y$  matrix, appending a new row for a new object and deleting rows for objects leaving discrimination. If object  $i$  is the object with the highest rate allotment and object  $n+1$  is being added, ERASE sets  $y_{n+1,k} = y_{ik} - \epsilon$  and renumbers the  $y_{ik}$  by rank. Thus the  $Y$  matrix gives a discrimination priority list.



#### 4. INTERCEPTOR ALLOCATION (MULTIPLE THRESHOLD FIRING DOCTRINE)

The interceptor allocation problem is to allocate the fixed stockpile of interceptors to all inbound threatening objects, consisting of warheads and decoys, so as to minimize the expected number of penetrating warheads. Expressed mathematically

$$\min_{(n_1, \dots, n_N)} \sum_{i=1}^N P_t(i; r_i, \text{history}) = \min_{(n_1, \dots, n_N)} \sum_{i=1}^N (1 - P_K(n_i)) PW_i$$

subject to the constraint  $\sum_{i=1}^N n_i \leq NI$ , where  $P_K(n)$  is the probability of kill given a net of  $n$  interceptors, the nominal net size  $n_i$  is the number of interceptors allocated to inbound object  $i$ ,  $PW_i$  is the probability that object  $i$  is a warhead (based on the discrimination returns),  $N$  is the number of objects, and  $NI$  is the number of interceptors available. It will be assumed in this discussion that the same  $P_K(\cdot)$  applies to all objects and that the allocation is done at a single point in time.

Given the discriminant returns, and hence the  $PW_i$ , this problem can be solved very easily if the  $P_K(\cdot)$  function is concave, as it proves to be. An elementary marginal return algorithm can then be used to allocate interceptors. This approach exactly uses all interceptors.

The optimal allocation of interceptors is characterized by a series of probability thresholds  $\{P_k\}$  where  $0 = P_0 < P_1 < \dots < P_{\text{net}+1} = 1$ , where "net" is the nominal net size to insure successful intercept. If object  $i$  has a discriminant return resulting in a value of  $PW_i$  satisfying  $P_k < PW_i < P_{k+1}$ , then it is optimal to send a partial net of size  $k$  to intercept that object.

A statistical description of the performance of this firing doctrine and a useful algorithm for selecting optimal thresholds can be obtained by treating the  $\{PW_i\}$  as random variables and transforming the constraint into an expected value constraint. It should be kept in mind that the engagement evaluation procedure retains a deterministic interceptor constraint as interceptor allocation decisions are actually made sequentially and each object processed is given the benefit of the remaining interceptors spread over the remaining objects.



The optimal set of thresholds  $P_k$  can be obtained by use of standard Lagrange multiplier techniques. The result of this optimization analysis yields the  $P_k$  having the form

$$P_k = \min\left\{1, \frac{\lambda}{P_K(k) - P_K(k-1)}\right\} \quad (k=1, \dots, \text{net}),$$

where  $P_0 = 0$  and  $\lambda$  is chosen to expend the available interceptors on an expected value basis.

This firing doctrine result can be transformed into a set of thresholds on any convenient axis. Thresholds  $\{T_k\}$ , on the axis of discriminant return, normalized to a scalar value, are thresholds chosen to match the probability thresholds using Bayes rule. If the densities of the discriminant returns are known in convenient form,  $\{T_k\}$  can be solved for in closed form. If only an operating characteristic curve ( $P_E$  vs.  $P_{FA}$ ) is known,  $\{T_k\}$  must be found numerically. The firing doctrine in threshold space is: Fire a partial net of size  $k$  to an object if its discriminant return is in  $(T_k, T_{k+1}]$ .

ERASE, and the defense, face a dynamic interceptor allocation problem and thus must modify the multiple threshold firing doctrine described above. The procedure postulated for the defense is as follows: At any point in the engagement when an interceptor allocation decision is needed for an object, allocate interceptors to the object in question as if to expend the remaining interceptors over the rest of the engagement. To do this, the defense estimates the number of RVs and decoys remaining and notes its actual interceptor stockpile remaining. This procedure is stable and exactly expends all interceptors by the end of the engagement, unless numerical errors adversely affect the computation.

At the user's option, ERASE approximates this postulated defense procedure. As a result of an approximation (noting expected interceptor stockpile remaining rather than all actual), to avoid calculating interceptor allocations for all possible interceptor usages, ERASE terminates with the probability of expending all interceptors generally in the 0.6 to 0.9 range.

## 5. A NOTE ON ERASE'S DISCRIMINATION MODEL

The evaluation of the  $PW_1$  and  $\{T_k\}$  or  $\{P_k\}$  mentioned earlier requires some knowledge of the discrimination model used by ERASE. A discriminant produces an output which may be interpreted as an estimate of an object's characteristic for some discriminants, e.g., length, slowdown drag, etc. or as an arbitrary index in other cases, e.g., pattern recognition. The discrimination measurements have random components. Thus the mean value (true signal) and the uncertainty or variance in the estimate (noise) are needed, and the uncertainty must be modeled as a function of the number of observations, observation time spacing, observation altitude, etc. The elementary characteristics of the discrimination process used by ERASE are:

$\Delta\mu$  = difference in mean value of the characteristic being measured, e.g., electrical length of the RV minus length of the decoy.

$\sigma_{\min}^2$  = the remaining variance that cannot be reduced by an arbitrarily large number of observations, e.g., variation in the characteristic that exists but which is not explainable by any defense model, for example, atmospheric or manufacturing variations.

$\sigma_{\text{un}}^2$  = the variance of a single observed measurement due to uncorrelated measurement noise, e.g., radar noise or trajectory related (crossing angle) uncertainty.

$\sigma_{\text{cor}}^2$  = the variance of a single observed measurement that is correlated from measurement to measurement, e.g., variation in the observable due to spin, precession, etc.

$T_{\text{cor}}$  = the decorrelation time for the exponentially autocorrelated uncertainty, i.e., the time needed for the autocorrelation in the discrimination output to drop to  $e^{-1}$ .

These variables may vary with altitude and discriminant employed. They may be different for the RV and decoy as well.



The overall net uncertainty (of standard deviation)  $\sigma$  is modelled as a function of number of observations and the observation rate, as these are the control variables ERASE uses in optimization. The model used by ERASE for  $\sigma$  in any fixed altitude region is:

$$\sigma^2 = \sigma_{\min}^2 + \frac{\sigma_{\text{un}}^2}{n} + \frac{\sigma_{\text{cor}}^2}{1+k(n-1)}$$

where  $n$  is the number of observations and  $k$  is a factor depending on the decorrelation time and observation rate. The value of  $k$  is given by

$$k = \frac{1 - \exp\{-1/rT_{\text{cor}}\}}{1 + \exp\{-1/rT_{\text{cor}}\}}$$

where  $r$  is the observation rate. The total variance,  $\sigma^2$ , is determined separately for the RV and decoy. The quality of the discrimination is given simply by the degree of separation of the distributions for the characteristic being measured for decoy and RV, i.e.,  $\Delta\mu/\sigma$ . If  $\Delta\mu/\sigma$  is large, the distributions are far apart and easily separable. It is  $\Delta\mu/\sigma$  values that are used in all threshold calculation, as this puts the distribution means conveniently at 0 for the decoy and 1 for the RV.

When an object receives discriminant measurements at various rates within an altitude band, a rule is needed to account for this. The procedure used in ERASE is to modify the correlated measurement variance term to  $\sigma_{\text{cor}}^2 / \{1 + k_1(n_1-1) + k_2n_2\}$  where  $(k_1, n_1)$  apply to the first measurement rate and  $(k_2, n_2)$  apply to the measurements taken at the second rate. When an object receives discriminant measurements in different altitude bands, the information in different altitude bands must be combined to yield an overall  $\sigma/\Delta\mu$  figure. ERASE uses one of two models for this combination, depending on the user's option, as follows:

$$(1) \left( \frac{1}{\sigma^2} \right)_{\text{combined}} = \sum \left( \frac{1}{\sigma^2} \right)_{\text{altitude bands}}$$

where  $\sigma^2$  for each altitude band separately is as given above with parameters appropriate for each (except that the denominator in the correlated noise expression in subsequent altitude bands is  $kn$  instead of  $1+k(n-1)$ ).

$$(2) (\sigma^2)_{\text{combined}} = (\sigma_{\text{min}}^2)_{\text{final altitude}} + (\sigma^2)_{\text{altitudes}}$$

when

$$\left( \frac{1}{\sigma^2} \right)_{\text{altitudes}} = \sum \left( \frac{1}{\sigma^2} \right)_{\text{altitude}} \quad \text{and}$$

$$(\sigma^2)_{\text{altitude}} = \frac{\sigma_{\text{un}}^2}{n} + \frac{\sigma_{\text{cor}}^2}{1+k(n-1)} .$$

These formulae are proper probabilistic statements if the discriminant information in separate altitude bands is uncorrelated.



## 6. SIMPLE ERASE EXAMPLE

To illustrate how ERASE performs its resource allocation through time, a simple BMD engagement was simulated. The results of this simulation are given below.

The attack consists of six objects which enter the search sector at intervals of .6 second. The first and fourth objects entering the search sector are warheads and the second, third, fifth and sixth objects are decoys. The objects enter the search sector at  $25.6^\circ$  elevation and a velocity of 22 kft/sec. This corresponds to a vertical descent rate of 9.5 kft/sec. The defense resources consist of 40 kw of radar power and six interceptors. The top of the search sector (chaff-clearing altitude) is 300 kft. Figure 6 depicts the situation.

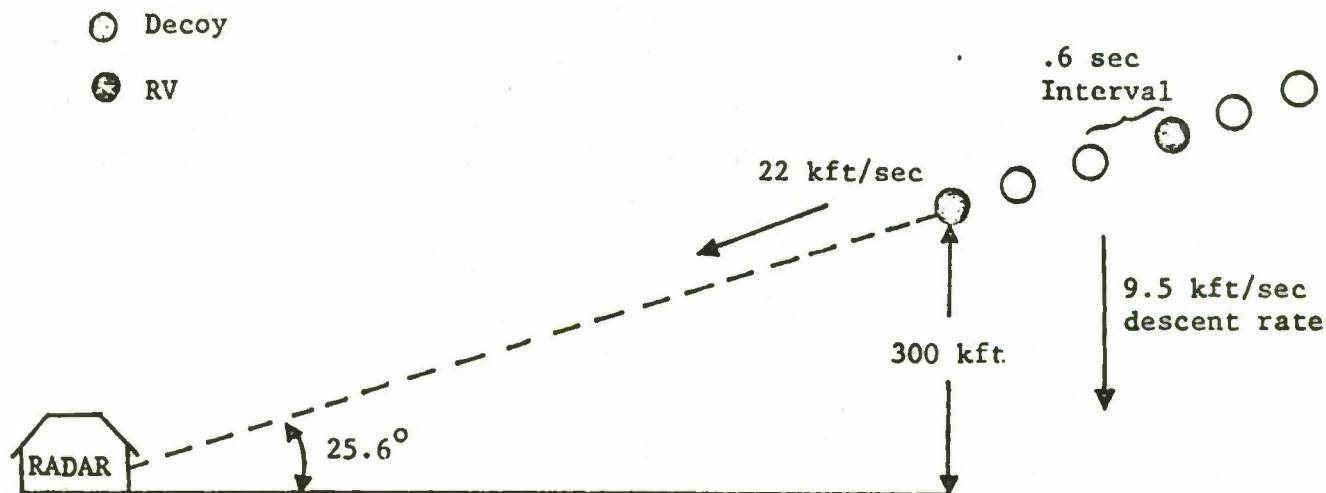


Figure 6

Table 1 contains a detailed breakdown of ERASE's decisions for this example. The first column is the battle time, with 0.00 seconds corresponding to the entry of object #1 into the search sector. The next six columns correspond to the six threat objects. Each entry in one of these columns consists of the current radar processing category, the current pulse rate assignment, and any tentative interceptor commit time if the object is in discrimination. The letter designations for the processing categories are:

- S - search
- V - verify
- ET - early track
- D - discrimination
- FT - final track.

The next five columns give a breakdown of the category allocation of radar resources. The upper number is the current total power allocated to that category, and the lower number is the current average pulse rate. Summing the upper numbers in these columns always gives the total radar power, 40 kw.

The radar function pulse energies and required pulse rates were assumed as follows: The search pulse energy for this example is 102.9 joules/pulse. The verify requirements are prespecified at 20.0 pulses/sec for .25 sec. The pulse energies for verify, early track, discrimination and final track are 102.9, 65.0, 575.1 and 65.0 joules/pulse. Early track has a minimum rate of 5 pulses/sec below which track would be lost. The rate for final track has also been prespecified at 5 pulses/sec. Discrimination mode also has this minimum pulse rate of 5 pulse/sec so that track will not be lost. It also has a maximum allowable rate of 40 pulses/sec chosen for convenience since higher pulse rates deliver little added information due to pulse to pulse correlation. In order to reduce the program run time, ERASE allocates discrete discrimination rates of 0, 5, 8, 15, 25 and 40 pulse/sec. (Realize that if a rate of 0 discrimination pulses/sec is specified, ERASE allocates 5 track pulses/sec in order to maintain track.) Any discrimination power left over after these discrete rates have been allocated is given to the object in discrimination for which

TABLE 1

TIME	OBJECT						TOTAL POWER AVERAGE RATE				
	1(RV)	2(DY)	3(DY)	4(RV)	5(DY)	6(DY)	S	V	ET	D	FT
0.00							40000. 388.7	0	0	0	0
0.62	V 20.0						37942.0 368.7	2058 20.0	0	0	0
0.73	V 20.0	V 20.0					35884.0 348.7	4116.0 20.0	0	0	0
0.87	ET 20.0	V 20.0					36642.0 356.1	2058.0 20.0	1300.0 20.0	0	0
0.92	D 22.15 CT 1.47	V 20.0					25204.6 244.9	2058.0 20.0	0	12737.4 22.15	0
0.97	D 22.789	ET 14.5					25953.5 252.2	0	940.4 14.5	13106.1 22.789	0
1.04	D 15.0	D 20.082					19824.4 192.7	0	0	20175.6 17.5	0
1.42	D 15.0 CT 1.42	D 20.082 CT 1.42					19824.4 192.7	0	0	20175.6 17.5	0
1.43	FT 5.0	FT 5.0					39350.0 382.4	0	0	0	650.0 5.0



TABLE 1 (Cont'd)

TOTAL POWER  
AVERAGE RATE

## OBJECT

TIME	1(RV)	2(DY)	3(DY)	4(RV)	5(DY)	6(DY)	S	V	ET	D	FT
1.69	FT 5.0	FT 5.0	V 20.0				37292.0 362.4	2058.0 20.0	0	0	650.0 5.0
1.81	FT 5.0	FT 5.0	V 20.0	V 20.0			35234.0 342.4	4116.0 20.0	0	0	650.0 5.0
1.94	FT 5.0	FT 5.0	ET 20.0	V 20.0			35992.0 349.8	2058.0 20.0	1300.0 20.0	0	650.0 5.0
1.99	FT 5.0	FT 5.0	D 21.775	V 20.0			24769.1 240.7	2058.0 20.0	0	12522.9 21.775	650.0 5.0
2.00	FT 5.0	FT 5.0	D 21.775 CT 2.83	V 20.0			24769.1 240.7	2058.0 20.0	0	12522.9 21.775	650.0 5.0
2.06	FT 5.0	FT 5.0	D 22.425	ET 14.2			25528.3 248.1	0	925.0 14.2	12896.7 22.425	650.0 5.0
2.13	FT 5.0	FT 5.0	D 15.00	D 19.52			19496.9 189.5	0	0	19853.1 17.3	650.0 5.0
2.63	FT 5.0	FT 5.0	D 19.52 CT 2.63	D 15.00 CT 2.63			19496.9 189.5	0	0	19853.1 17.3	650.0 5.0
2.64	FT 5.0	FT 5.0	FT 5.0	FT 5.0			38700.0 376.1	0	0	0	1300.0 5.0
3.16	FT 5.0	FT 5.0	FT 5.0	FT 5.0	V 20.0		36642.0 356.1	2058.0 20.0	0	0	1300.0 5.0



TABLE 1 (Cont'd)

TIME	OBJECT						TOTAL POWER AVERAGE RATE					
	1(RV)	2(DY)	3(DY)	4(RV)	5(DY)	6(DY)	S	V	ET	D	FT	
3.41	FT 5.0	FT 5.0	FT 5.0	FT 5.0	ET 20.0		37400.0 363.5	0	1300.0 20.0	0	1300.0 5.0	
3.46	FT 5.0	FT 5.0	FT 5.0	FT 5.0	D 22.583		25712.5 249.9	0	0	12987.5 22.583	1300.0 5.0	
3.47	FT 5.0	FT 5.0	FT 5.0	FT 5.0	D 22.583 CT 4.21		25712.5 249.9	0	0	12987.5 23.583	1300.0 5.0	
4.02	FT 5.0	FT 5.0	FT 5.0	FT 5.0	D 22.583 CT 4.30		25712.5 249.9	0	0	12987.5 22.583	1300.0 5.0	
4.14	FT 5.0	FT 5.0	FT 5.0	FT 5.0	D 21.402	V 20.0	24333.6 236.5	2058.0 20.0	0 -	12308.4 21.402	1300.0 5.0	
4.25	FT 5.0	FT 5.0	FT 5.0	FT 5.0	D 21.402 CT 4.25	V 20.0	24333.6 246.5	2058.0 20.0	0	12308.4 21.402	1300.0 5.0	
4.26	FT 5.0	FT 5.0	FT 5.0	FT 5.0	FT 5.0	V 20.0	36317.0 352.9	2058.0 20.0	0	0	1625.0 5.0	
4.39	FT 5.0	FT 5.0	FT 5.0	FT 5.0	FT 5.0	ET 20.0	37075.0 360.3	0	1300.0 20.0	0	1625.0 5.0	

TABLE 1 (Concluded)

TIME	OBJECT						TOTAL POWER AVERAGE RATE				
	1(RV)	2(DY)	3(DY)	4(RV)	5(DY)	6(DY)	S	V	ET	D	FT
4.44	FT	FT	FT	FT	FT	D	25494.8	0	0	12880.2	1625.0
	5.0	5.0	5.0	5.0	5.0	22.397	247.8			22.397	5.0
4.50	FT	FT	FT	FT	FT	D	25494.8	0	0	12880.2	1625.0
	5.0	5.0	5.0	5.0	5.0	22.397 CT 5.01	247.8			22.397	5.0
4.87	FT	FT	FT	FT	FT	D	25494.8	0	0	12880.2	1625.0
	5.0	5.0	5.0	5.0	5.0	22.397 CT 4.87	247.8			22.397	5.0

the information return would be greatest (this is usually the object which has been in discrimination the least amount of time).

The category allocation performed at time 2.06 seconds is typical for this example. Objects #1 and #2 are in final track with 5 pulses/sec each at 65.0 joules/pulse for a total of 650.0 watts of radar resource. Object #3 is in discrimination and must receive a minimum of 5 pulses/sec at 65.0 joules/pulse for a total of 325.0 watts. (These are final track pulses in order to maintain track.) Object #4 is in early track and must receive a minimum of 5 pulses/sec at 65.0 joules/pulse for a total of 325.0 watts. There is also a minimum search rate of 5 pulses/sec at 102.9 joules/pulse for a total of 514.5 watts. These minimum necessary rates total 1814.5 watts of power. The category allocator then allocates the remaining power, 38185.5 watts. This power is allocated by minimizing the processing time for the average object:

$$T = \frac{481.13}{5 + \Delta X_S} + \frac{1}{5 + \Delta X_{ET}} + \frac{20.88}{\Delta X_D}$$

to be minimized with respect to  $\Delta X_S$ ,  $\Delta X_{ET}$  and  $\Delta X_D$  subject to the power constraint

$$(102.9)\Delta X_S + (65.0)\Delta X_{ET}(2) + (575.1)\Delta X_D = 38185.5 .$$

This results in  $\Delta X_S = 243.1$ ,  $\Delta X_{ET} = 9.2$  and  $\Delta X_D = 22.4$ .

Multiplying each rate by its appropriate pulse energy and adding to each the minimum power set aside for that category yields

$$R_S = 25528.3, R_{ET} = 925.0 \text{ and } R_D = 12896.7.$$

To compute the search pulse rate,  $R_S$  is divided by the search pulse energy, and to compute the early track rate for each object,  $R_{ET}$  is divided by the product of the pulse energy and number of objects in early track.



For time = 2.06 sec, there is only one object in discrimination and its pulse rate is as given, 22.4 pulses/second. At time 2.06,  $X_s = 248.1$ ,  $X_{ET} = 14.2$ , and  $X_D = 22.4$ .

Table 2 gives a summary of the interceptor commitments for each object. The prelaunch and launch reliability was assumed to be 90%, thus of the six interceptors a binominally distributed random number are assumed to be operational. ERASE allocates interceptors so that the expected number allocated during the course of the engagement is approximately equal to the expected number of operational interceptors, i.e., 5.400.

The mean number of interceptors available for intercept is 5.400 due to launch reliability considerations. ERASE attempts to expend this number of interceptors on an expected value basis. The actual probabilities of firing 0-4 are slightly different because the defense is never allowed to expend more interceptors than it actually has. Thus, although ERASE wanted to fire 5.397 interceptors, shortages that arise in some combinations of outcomes would result in sending only 5.032 on the average, with the balance, 0.368, expected to be remaining after the end of the engagement. The probability of expending all interceptors was 0.698. (The defense would do better if the superior but more expensive algorithm were used.)

The discrimination power allocation at time 2.63 sec. is the only allocation at which 2 objects contend for the discrimination power. A summary of the calculations to allocate this power between objects #3 and #4 is given in Table 3. The three parts of this table give the intermediate calculations needed to effect a discrimination power allocation. First, the probability of penetration must be calculated for each object for each discrimination rate allowed for various commit times. The results of these calculations are in part (a) of the table. From the minimal probability of penetration for each rate, part (b) can be calculated, obtaining the marginal gain in performance per watt of power expended. Note that increasing the power/pulse rate reduces the probability of penetrating, hence the gains are negative numbers. Power is allocated based on the rank order of marginal gains found in (b): this is done in part (c). The power allocation in part (c) proceeds until the power allocated to discrimination by the category allocator is exhausted.



TABLE 2  
PROBABILITY OF INTERCEPTOR NET SIZE

OBJECT	COMMIT TIME	0	1	2	3	4 OR MORE	DESIRED EXPECTED NUMBER OF INTERCEPTORS
#1(RV)	1.43	.005	.010	.280	.705	0	2.685
#2(DY)	1.43	.996	.004	0	0	0	.004
#3(DY)	2.64	.994	.003	.003	0	0	.009
#4(RV)	2.64	.002	.005	.290	.703	0	2.694
#5(DY)	4.26	.999	.001	0	0	0	.001
#6(DY)	4.88	.996	.004	0	0	0	.004
EXPECTED NO. FIRED DURING ENGAGEMENT							5.397

TABLE 3

## DISCRIMINATION ALLOCATOR SUMMARY

(a) Probability of Penetration Tables

<u>Object #3</u>	<u>Discrimination Rate</u>					
<u>Time</u>	0	5	8	15	25	40
$T_o$	.279*	.274*	.274*	.273*	.273*	.273*
$T_o + \Delta T$		.282	.282	.282	.282	.282
$\vdots$						
$T_o + 5\Delta T$	<u>(NOT CALCULATED AS PROBABILITY OF PENETRATION INCREASING)</u>					
Last Ditch	.466	.466	.466	.466	.466	.466
Best Time	$T_o$	$T_o$	$T_o$	$T_o$	$T_o$	$T_o$

<u>Object #4</u>	<u>Discrimination Rate</u>					
<u>Time</u>	0	5	8	15	25	40
$T_o$	.288*	.287*	.287*	.286*	.286*	.286*
$T_o + \Delta T$		.296	.296	.296	.296	.296
$\vdots$						
$T_o + 5\Delta T$	<u>(NOT CALCULATED AS PROBABILITY OF PENETRATION INCREASING)</u>					
Last Ditch	.466	.466	.466	.466	.466	.466
Best Time	$T_o$	$T_o$	$T_o$	$T_o$	$T_o$	$T_o$

(\* = minimum probability of penetration for each rate)

(b) Marginal Gain Tables

<u>Pulse Rate Interval</u>	<u>Object #3</u>	<u>Object #4</u>	<u><math>\Delta</math> Probability of Penetration</u> <u><math>\Delta</math> Power Used</u>
0-5	$-1.95 \times 10^{-6}$	$-4.28 \times 10^{-7}$	
5-8	$-1.41 \times 10^{-7}$	$-3.49 \times 10^{-8}$	
8-15	$-4.67 \times 10^{-8}$	$-1.48 \times 10^{-8}$	
15-25	$-1.18 \times 10^{-8}$	$-3.76 \times 10^{-9}$	
25-40	$-5.50 \times 10^{-9}$	$-1.40 \times 10^{-9}$	

(c) Power Allocation to Objects

<u>Object</u>	<u>Rate</u>	<u><math>\Delta</math> Prob/<math>\Delta</math> Power</u>	<u>Power Used</u>	<u>Commit Time</u>
3	0	-	325.0	2.63
4	0	-	325.0	2.63
3	0→5	$-1.95 \times 10^{-6}$	2550.6	2.63
4	0→5	$-4.28 \times 10^{-7}$	2550.6	2.63
3	5→8	$-1.41 \times 10^{-7}$	1725.3	2.63
3	8→15	$-4.67 \times 10^{-8}$	4025.8	2.63
4	5→8	$-3.49 \times 10^{-8}$	1725.3	2.63
4	8→15*	$-1.48 \times 10^{-8}$	4025.8	2.63*
3	15→19.52*	$-1.18 \times 10^{-8}$	<u>2599.7</u>	2.63*

19,853.1 watts

(\* = scheduled commit times and final power allocations)





DEPARTMENT OF THE AIR FORCE  
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REPLY TO  
ATTN OF SULR (Diane Corazzini, 4553)

2 May 1985


SUBJECT Approval of Reports to be Downgraded to "Statement A"

TO AFGL/SULL  
Research Library

1. The following reports have been approved by the Public Affairs Office (ESD/PAM) for downgrading to Statement A:

<u>ESD-TR-77-270</u>	<u>ERASE: An Overview</u>
ESD-TR-77-30	Theory and Operating Characteristics of TRAPATT Amplifiers
ESD-TR-77-69	Space Communications
ESD-TR-77-122	Space Communications
ESD-TR-77-229	Space Communications

2. ESD-TR-77-348 "LES-8/9 Antenna Systems. Vol 2: S-Band Telemetry" has been approved on the condition that a reference be removed. Attached is page iii, with the reference deleted, enabling this report to now be labelled "Statement A."

  
DIANE CORAZZINI  
Research Publications

1 Atch  
Corrected page for ESD-TR-77-348